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The optimal design of Ponzi schemes in finite economies ☆

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Abstract

As no rational agent would be willing to take part in the last round in a finite economy, it is difficult to design Ponzi schemes that are certain to explode. This paper argues that if agents correctly believe in the possibility of a partial bailout when a gigantic Ponzi scheme collapses, and they recognize that a bailout is tantamount to a redistribution of wealth from non-participants to participants, it may be rational for agents to participate, even if they know that it is the last round. We model a political economy where an unscrupulous profit-maximizing promoter can design gigantic Ponzi schemes to cynically exploit this "too big to fail" doctrine. We point to the fact that some of the spectacular Ponzi schemes in history occurred at times where and when such political economies existed—France (1719), Britain (1720), Russia (1994), and Albania (1997).

JEL classification: G10; G18; F36; P20

Keywords: Ponzi schemes; Bubbles; Bailout; Moral hazard

> *At the height of his success in Boston in* 1920, *Charles A. Ponzi was hailed by those he was cheating as the greatest Italian who ever lived. "You're wrong," he said modestly, "there's Columbus, who discovered America, and Marconi, who discovered radio." "But, Charlie, you discovered money," they told him.*
>
> *From an article in the San Diego Daily Transcript* (7/16/1974)

The money-making machine that Charles A. Ponzi invented in Boston in June 1919 was elegant in its simplicity. It had three critical components. First, he convinced a group

☆ A previous version of this paper was titled "On the possibility of Ponzi schemes in transition economies."

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of people about an investment idea: coupons issued by the International Postal Union seemingly violated the law of one price and, therefore, offered an arbitrage opportunity. Second, he promised them a high return on their investment: a 50 per cent interest every ninety days. And third, he built credibility by initially delivering on his promises: "interest plus principal" of the earlier "investments" was paid by money "invested" by those who were recruited into the scheme later. As his reputation spread by word-of-mouth, people flocked from all over New England to invest. Ponzi took in about $200,000 a day. The scheme finally crashed when the Boston Globe exposed him in August 1920.[1]

Such types of schemes have existed before Ponzi and continue to exist after him.[2] The first extensively recorded scheme, covered by Mackay (1841), was conceived by a Scotsman, John Law, in France in 1719. It was immediately followed by the South Sea Bubble in Britain in 1720. Today, thanks to the Internet, Ponzi schemes are making a dramatic comeback.[3]

Economists have long been puzzled by Ponzi schemes because they seemingly violate the laws of rationality. An extensive literature has developed to analyze the conditions under which Ponzi schemes and other types of bubbles can arise in economies that go on forever.[4] The existence of these conditions insure that it is rational for agents to participate in any round because they expect to close their position in a later round at a gain. So the Ponzi scheme goes on forever. But the unanswered question is: How can Ponzi schemes survive in economies that do not go on forever? This is the question this paper addresses.

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[1] This description of the original Ponzi scheme is summarized from an article appearing in the San Diego Daily Transcript on July 16, 1974. Ponzi schemes, though sharing the same pyramidal structure as chain letter schemes and pyramid schemes, are different. The fundamental difference is that in a Ponzi scheme a promoter, like Ponzi, has considerable control over every aspect of the scheme, including when to terminate the scheme. This allows the promoter to make money from every round. The initiators of chain letter schemes and pyramid schemes, on the other hand, do not have control and they make little money, if at all, after the first round. In chain letter schemes, a recruit receives a letter with a list of names on it, and is asked to send a sum of money, x, to the name at the top of the list, and then eliminate this name and add his name at the bottom. The recruit is then asked to mail copies of the letter and the instructions to n more recruits. Hopefully, these n new recruits will follow the same procedure, and the process will continue, and the old recruit's name will gradually move to the top of the list, and he will receive a lot of money. In a pyramid scheme, a recruit is asked to give a sum of money, x, to a recruiter, and then is asked to enlist n more recruits and collect x from each one of them.

[2] For an excellent introduction to Ponzi schemes as well as to many other types of bubbles, see Garber (1990). For detailed expositions, the classics by Mackay (1841) and Kindleberger (1978) are recommended.

[3] This may be because word-of-mouth referrals have been substituted by the more efficient click-of-mouse referrals. In 1997, the United States Federal Trade Commission launched "Operation Missed Fortune," a federal and state crackdown on fraudulent Ponzi schemes. Even law enforcement officers were not spared; 67 employees of the Sacramento Police Department were being investigated. The FTC issued an alert: US citizens are being asked to report Ponzi and other get-rich-quick schemes to the National Fraud Information Center at 1-800-876-7060 or via the Internet at http://www.fraud.org. In 1998, the Chinese government banned all businesses that employed some elements of Ponzi schemes, completely disrupting the selling operations of even legitimate businesses (see Wall Street Journal, May 1, 1998, editorial titled "Avon Ladies Under Siege.")

[4] A partial list of papers in the bubbles literature would be Brock (1979, 1982), Bewley (1980), Tirole (1982, 1985), Scheinkman (1988), Gilles and LeRoy (1992), Kocherlakota (1992), and Huang and Werner (1997). A recent paper by Santos and Woodford (1997) comprehensively covers this area, and shows that the conditions under which rational bubbles are possible are fragile.

To explain Ponzi schemes in economies that do not go on forever, additional assumptions have been introduced to justify why agents in the last round would take part. This literature can be broadly classified into two strands. The first strand of papers is behavioral, and it assumes that some agents are irrational.[5] The second strand of papers, maintaining neo-classical assumptions, have all introduced asymmetric information plus a few other reasonable assumptions to obtain their results.[6] For example, in Abreu and Brunnermeier (2001), dispersion of opinion causes dispersion of exit strategies of rational arbitrageurs, but these rational arbitrageurs cannot temporarily coordinate their selling strategy to burst the bubble. Our paper belongs to this second strand. What differentiates our paper from this literature is that in our paper we assume symmetric information. All the participants in the Ponzi scheme *know exactly* when the scheme started, which round is being played, and when it will end. All the participants are identical and all are rational. Though the promoter of the Ponzi scheme starts the scheme with a good investment idea, the participants take part not because they believe in the idea, but because it is rational for them to take part. As far as we know, ours is the first paper in the literature to demonstrate the existence of Ponzi schemes in a finite economy where all agents are identical, rational, and have symmetric information.

We argue in this paper that if agents correctly believe in the possibility of a partial bailout when a gigantic Ponzi scheme collapses, and they recognize that a bailout is tantamount to a redistribution of wealth from non-participants to participants, it may be rational for agents to participate, even if they know that it is the last round. We model a political economy where an unscrupulous profit-maximizing promoter can design gigantic Ponzi schemes to cynically exploit this “too big to fail” doctrine. The contribution of this paper is to precisely lay out the finite political economy where Ponzi schemes can occur; to detail the characteristics of the Ponzi scheme that will arise in such an economy; and then to link our theory and its implications to some spectacular Ponzi schemes that have occurred in history.

In a classic Ponzi scheme, a promoter sells certificates to citizens in each round, promising them an attractive return per round on their investment. Since the money raised in a round is partially used to pay off the obligation of investors from a previous round, the revenue of the promoter comes from the sales in the initial round plus a fraction of the sales from later rounds. Also, as a record of successful payment develops and information about the fantastic scheme spreads by word-of-mouth (or click-of-mouse), most of the costs of the promoter are the marketing costs of reaching the initial group of citizens. The risk-neutral promoter designs the scheme to maximize expected profits. He has to take two things into account. First, he has to insure that both he and the citizens have the incentives to participate in each round till the very last planned round. Second, he needs to take into account the fact that there is a regulator who may terminate the scheme at any

[5] Shiller (1981), Shleifer and Summers (1990), and De Long et al. (1990) have explained bubbles using this assumption.

[6] See Abreu and Brunnermeier (2001), Allen and Gorton (1993), Allen et al. (1993), Bhattacharyya and Lipman (1995) for examples of rational bubbles in economies where the number of rounds played and/or time is finite.

point in time, and the probability of regulatory intervention is negatively correlated with his political connectedness.

The economic forces at work in the last round, as one might expect, are very different from the other rounds. The promoter plans to terminate the scheme in this round, if the scheme has not already been terminated before by the regulator. The citizens who are affected are the participants in the last round, because they will not recover their money. They organize to use the state's assets for a bailout, and as their size is large, and the size of the state assets is large, the probability of this bailout is not zero. As our subsequent discussion will reveal, partial bailouts from state assets have happened for some failed Ponzi schemes in history. As every citizen has an equal claim on the state assets, the bailout amounts to a redistribution of wealth from non-participants to participants. The parameters of the Ponzi scheme are set such that the expected loss incurred by participating, which is the lost investment minus the expected net redistribution gain from the bailout, is not greater than the loss incurred by not participating, which is the expected redistribution loss from the bailout. So the citizen takes part in this last round.

The economic forces at work in the other rounds are as follows. We first address why the promoter has to continue his scheme till his last planned round. The answer is that at every round, the promoter has to trade off the sure revenue he will get if he terminates now against the expected revenue he will get if he terminates a round later. The parameters of the Ponzi scheme are set to insure that the latter expected revenue is greater than or equal to the sure former revenue in all rounds except the last round, and it is lower in the last round. In other words, the Ponzi scheme is subgame-perfect. The parameters of the Ponzi scheme are also set to insure that in the initial rounds the participating risk-neutral citizen's expected loss if the regulator intervenes is less than the expected gain the participating citizen achieves if the regulator does not intervene. So the citizen takes part in the initial rounds as well.

We then go on to characterize the finite political economy where the above constrained maximization problem of the promoter has a solution. We find the following to be the features of this finite political economy: a large public sector (the proportion of national wealth owned by the state is above a lower bound), ambiguous laws governing the transfer of property rights from the state to the citizen (victims of a failed Ponzi scheme may organize to use the state's assets for a bailout, the probability of which to occur is above a lower bound), political connections (the probability of early termination of the Ponzi scheme by a regulator is below an upper bound), an inexpensive access to citizens through mass media (advertising effectiveness is above a lower bound), and a low penalty for the offending promoter (penalty is below an upper bound).

It may not be mere coincidence that some of the biggest Ponzi schemes in history have occurred at times where and when such political economies existed—France (1719), Britain (1720), Russia (1994), and Albania (1997).[7]

A reading of Mackay's (1841) account of the Mississippi Scheme indicates that John Law's scam had the blessings of France, a state whose finances were in a mess after the death of Louis XIV. According to Mackay, "He proposed to the regent (who could refuse

[7] It should be noted here that some other episodes—the Tulipmania in Netherlands in 1636–1637 and Ponzi's exploits in Boston in 1920—did not have partial bailouts or promises of partial bailouts. So the hypothesis forwarded in this paper cannot explain all Ponzi schemes.

him nothing) to establish a company that should have the exclusive privilege of trading to the great river Mississippi and the province of Louisiana." In 1719, Law's company, the Compagnie des Indes, was further granted the exclusive privilege of trading with the East Indies, China, and the South Seas. John Law started his scheme that year. His scam had the three critical ingredients of a classic Ponzi scheme. First, there was an investment idea: a share in the profits that were to be made by trade with exotic lands. Second, an attractive return was promised: a 40% annual return on the shares of the Mississippi Company. And third, obligations were initially met: he delivered an annual return of 120% in the beginning. However, it had one feature that we did not observe in Charles Ponzi's scheme—intimate involvement by the ruling class. It should also be noted that when the scheme came crashing down, the holders of useless Mississippi stock were given 2.5% interest-bearing notes that were *secured* by the municipal revenues of the city of Paris.[8]

The South Sea Bubble in Britain in 1720 was, as Garber (1990) aptly describes, a shadow of the Mississippi Scheme. The Whig ministry had been dismissed, and public debt was at an astounding ten millions sterling. In 1720, Parliament granted the South Sea Company monopoly rights over trade with the South Seas and, in exchange, obtained attractive refinancing terms for the state debt. The South Sea Company then acted like Law's company: issue successive rounds of stock that promised a share of trading profits, deliver initial attractive returns (100% return from February to April 1720), and then disintegrate. Parliament partially bailed out investors by *writing off* 7.1 million sterling of the company's debt.[9]

History repeated itself in Russia in 1994 as tragedy. The MMM scheme promoted by Sergei Mavrodi collapsed. He had promised annual returns of 2000%, recruited 5 million Russians, and had become the sixth richest man in Russia. When the scheme collapsed, Mavrodi promised a government bailout of MMM if he was elected to the Russian Duma, and he did get elected. That saved him from criminal prosecution.[10] A notable feature of this scheme was the initial non-discouragement by the regulators and a *possibility of a partial bailout* after the collapse.[11] Bailouts, however, were not promised to upset citizens of the myriad smaller Ponzi schemes (like Tibet, Russki Dom Selenga and Khopor) that had also sprouted.

These themes were replayed in a smaller scale in other transition economies in the 1990s. Ponzi schemes were reported in Romania: there were about 600 schemes, the biggest of which was Caritas which involved twenty percent of the population and promised a 800 percent return in 100 days. Bulgaria, Slovakia, Serbia, and the Czech Republic were the other countries where Ponzi schemes appeared.

Then, in 1997, history repeated itself in Albania as farce. Maksude Kademi, Bakshim Driza, and Rapush Xhaferi had promised returns as high as 100% in six months and

[8] The observations in this paragraph come from Mackay's (1841) account.

[9] The observations in this paragraph come from Garber's (1990) account.

[10] This author has been following the career of Sergei Mavrodi with great interest. His latest venture is another Ponzi scheme, this time on the internet. On July 9, 2000, the US Securities and Exchange Commission brought a suit against Mavrodi's Dominican Republic-registered company called Stockgeneration.

[11] Pallada Asset Management in Moscow was entrusted with the responsibility of managing a federal fund setup to compensate victims of Ponzi schemes (Reuters, February 26, 1997).

had sold their certificates to about half the population. They had attracted a sum which was about four times Albania's state budget, twice its bank deposits, and roughly equal to its GDP. When their "foundations" collapsed, about a sixth of the population lost all their savings, and violent civil unrest erupted. A salient feature of the three Albanian schemes was the role of the ruling class. State TV actively promoted these funds, giving the impression of official approval. Political parties *endorsed* them. Election posters often included the logos of the funds. Finally, when the schemes collapsed, the government accepted "moral responsibility" *to pay back* at least some of the $370 million lost. This was a large sum compared to the annual state budget of $500 million.[12]

The paper is organized as follows. Section 2 sets up the optimization problem of an unscrupulous promoter of a Ponzi scheme, who wants to exploit a citizen's belief that a partial bailout is likely if a critical mass of citizens are adversely affected. Section 3 characterizes the conditions for a solution. Section 3 concludes with some sobering policy implications.

1. The model

1.1. The political economy

Let the total wealth of a nation, of which a significant fraction is owned by the state, be normalized to unity. The property rights to the wealth owned by the state could be transferred to its own citizens. Let the proportion of national wealth that may be so transferred be α.

Citizens of this nation are modeled as a continuum of risk-neutral individuals whose mass is normalized to unity. This assumption insures that no citizen is pivotal enough to act strategically. Each citizen has an equal claim on the state assets. However, if due to any circumstance, a privileged mass M of citizens ($M < 1$) usurps these claims, the net loss per unit mass of citizens excluded from this privileged group is α, and the net gain per unit mass of citizens included in this privileged group is $\alpha(1/M - 1)$.[13]

Citizens do not have a representative government. Laws governing the transfer of property rights from the state to its citizens are unclear. So it is possible that all citizens may not get their fair share of the state assets. Such an event might happen if a politically powerful clique gets more than its fair share.[14] Or such an event might happen if a significant fraction of the populace unfortunately discover that they are not being given

[12] The information about the Ponzi schemes in the transition economies of the 1990s is culled from news reports in CNN News, Radio Free Europe, and various issues of Time Magazine. The facts were independently verified by Sadiraj in Albania (see Sadiraj et al. (1998)) and by Mikhalev in Moscow. Figure 1 shows a copy of a certificate used by MMM in Russia; Fig. 2 shows a copy of a certificate used by the foundation Gjallica in Albania. Though we will discuss some salient features of these certificates later in the paper, interested readers may contact the author for precise translations. In Albania, though the "foundations" were pure Ponzi schemes, "investment companies" like Vega did have some legitimate economic investments.

[13] There is an implicit "common value" assumption here of the state assets. A "private value" assumption would make the analysis needlessly complicated without adding much insight.

[14] Boycko and Shleifer (1993) discuss how special privileges had to be given to managers, workers, and local governments in the earlier stages of Russian privatization.

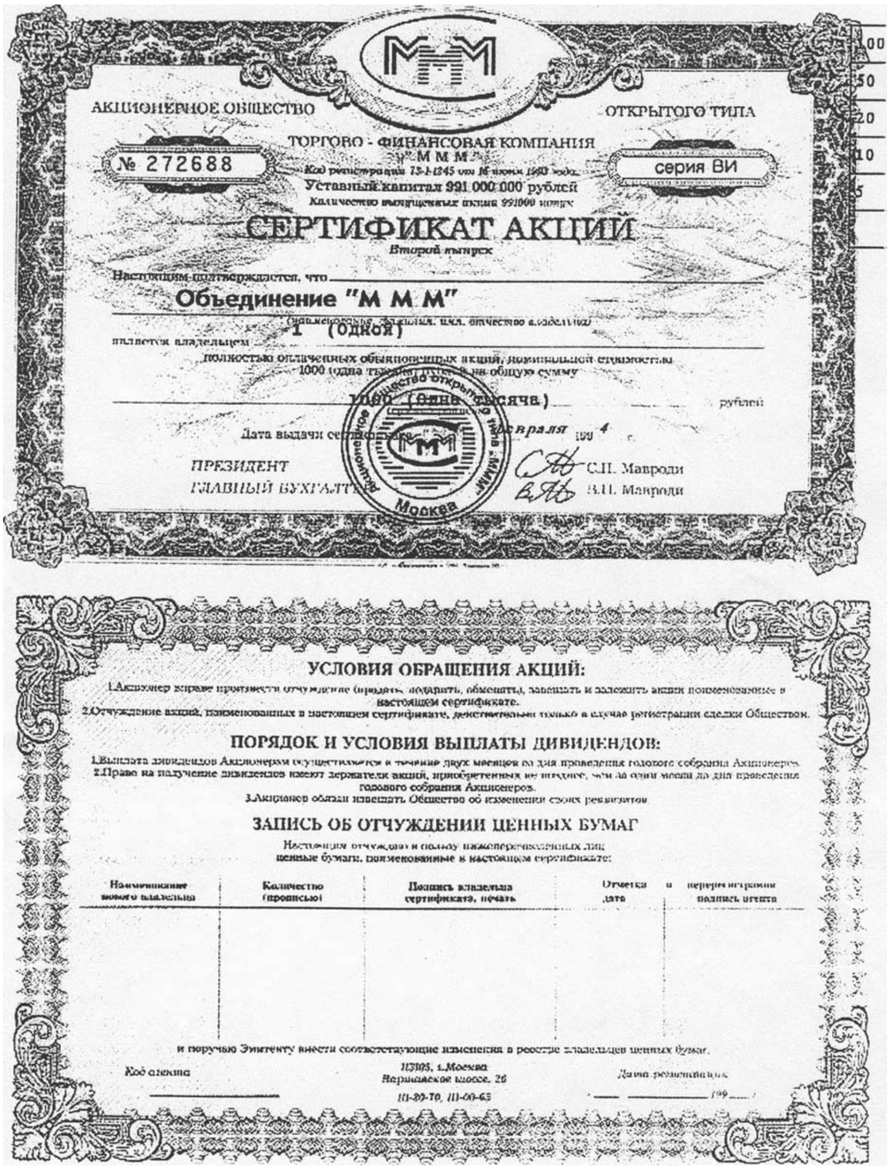

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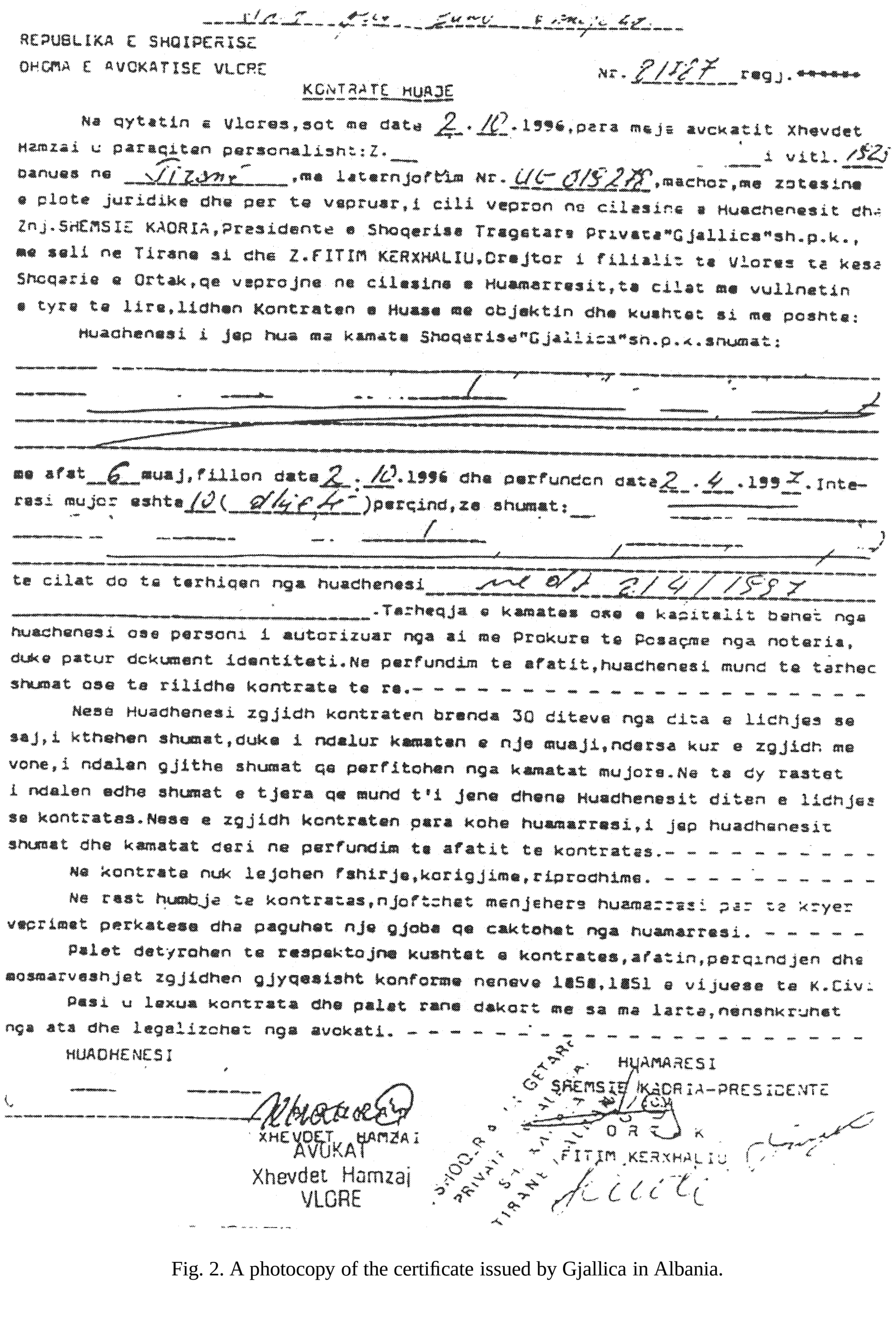
Fig. 1. A photocopy of the front and back of certificate issued by MMM in Russia.

what has been promised to them in a Ponzi scheme, and demand compensation from the state.[15]

[15] As discussed in the introduction of this paper, this did occur in France (1719) and Britain (1720), and may occur in Russia (1994) and Albania (1997). It is interesting to note that Ponzi's victims received no compensation from the United States in 1920. The victims of Tulipmania in Netherlands received no state compensation in 1637.



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KONTRATE HUAJE

Ne qytetin e Vlores, sot me date 2.10.1996, para meje avokatit Xhevdet Hamzai u paraqiten personalisht: Z. ______ i vitl. ______ banues ne ______, me leternjoftim Nr. ______, madhor, me zotesine e plote juridike dhe per te vepruar, i cili vepron ne cilesine e Huadhenesit dhe Znj. SHEMSIE KADRIA, Presidente e Shoqerise Tregetare Private "Gjallica" sh.p.k., me seli ne Tirane si dhe Z. FITIM KERXHALIU, Drejtor i filialit te Vlores te kesaj Shoqerie e Ortak, qe veprojne ne cilesine e Huamarresit, te cilat me vullnetin e tyre te lire, lidhen Kontraten e Huase me objektin dhe kushtet si me poshte:

Huadhenesi i jep hua me kamate Shoqerise "Gjallica" sh.p.k. shumat:

me afat 6 muaj, fillon date 2.10.1996 dhe perfundon date 2.4.1997. Interesi mujor eshte 10 (______) perqind, ze shumat: ______

te cilat do te terhiqen nga huadhenesi ______ 2/4/1997

______. Terheqja e kamates ose e kapitalit behet nga huadhenesi ose personi i autorizuar nga ai me Prokure te Posaçme nga noteria, duke patur dokument identiteti. Ne perfundim te afatit, huadhenesi mund te terheqe shumat ose te rilidhe kontrate te re.- - - - - - - - - - - - - - - - - - -

Nese Huadhenesi zgjidh kontraten brenda 30 diteve nga dita e lidhjes se saj, i kthehen shumat, duke i ndalur kamaten e nje muaji, ndersa kur e zgjidh me vone, i ndalen gjithe shumat qe perfitohen nga kamatat mujore. Ne te dy rastet i ndalen edhe shumat e tjera qe mund t'i jene dhene Huadhenesit diten e lidhjes se kontratas. Nese e zgjidh kontraten para kohe huamarresi, i jep huadhenesit shumat dhe kamatat deri ne perfundim te afatit te kontratas.- - - - - - - - - -

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Ne rast humbje te kontratas, njoftohet menjehere huamarresi per te kryer veprimet perkatese dhe paguhet nje gjobe qe caktohet nga huamarresi. - - - - -

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Pasi u lexua kontrata dhe palet rane dakort me sa me larte, nenshkruhet nga ata dhe legalizohet nga avokati. - - - - - - - - - - - - - - - - - - -

HUADHENESI

HUAMARESI
SHEMSIE KADRIA-PRESIDENTE

FITIM KERXHALIU

XHEVDET HAMZAI
AVUKAT
Xhevdet Hamzaj
VLORE

Fig. 2. A photocopy of the certificate issued by Gjallica in Albania.

We model this situation as follows. When a Ponzi scheme is operating, it is in the public interest to terminate it. However, as the citizens are atomistic, and there is no representative government to look after the public interest, there is a coordination failure. There exists, however, a regulator who can stop the scheme with a probability θ. This regulator is a left-over from the previous regime. He trades off public welfare with the private welfare of a ruling class to which the promoter of the Ponzi scheme belongs. A simple way to model this in a political economy is to assume that the regulator's utility function is [$\theta *$ Public Welfare $+ (1-\theta) *$ Private Welfare of Ruling Class], where $\theta \in [0,1]$. θ could be interpreted as the weight the regulator assigns to public welfare, or a measure of the regulator's honesty, or the lack of political connections of the promoter. The higher is θ, the more important is the public welfare to the regulator, or the more honest is the regulator, or the less is the political connection of the promoter.

If the Ponzi scheme explodes, the affected citizens organize to use the state's assets, α, for a bailout. As the size of the upset citizens is significant, and the size of the state assets is large, there exists a possibility that this might happen. Let n_a be the mass of citizens adversely affected, and $p(n_a)$ be the resultant probability of bailout. We study the following political model: there is no bailout if n_a is less than a critical mass, n^*, and bailout with probability β if n_a is greater than or equal to n^*. Here $1 > n^* \geqslant 0.5$. This formalization covers a wide range of political regimes. It covers situations where decisions about a bailout are made by the majority vote. For simple majorities, we need $n^* = 0.5$, and for super majorities, we need $n^* > 0.5$. It covers regimes where minority rights are not respected ($\beta = 1$) or somewhat respected ($0 < \beta < 1$).[16]

Throughout this paper, we assume that the size of the state's assets used for the bailout, α, is a constant. It could be argued that the bailout size will depend on the numerical strength of the aggrieved citizens, i.e., α is an increasing function of n_a. As we will see later, this generalization is possible, as long as $\alpha(n_a)$ does not increase too fast with respect to n_a.[17] The reason is that if $\alpha(n_a)$ increases too fast with respect to n_a, the bailout per person will increase as the number of persons involved increases, which implies that the promoter would like to involve everyone. That cannot lead to a rational Ponzi scheme in our model, because in a rational Ponzi scheme we need non-participants. The reason we need non-participants is because the fundamental insight of our paper is that it may be rational for agents to participate in the last round of a Ponzi scheme that is going to explode immediately afterwards, since everyone realizes that the ensuing bailout will cause a redistribution of wealth from non-participants to participants.

[16] Throughout this paper, we assume that a bailout is carried out by transferring state assets to the aggrieved citizens. Bailouts could also be achieved by raising state revenue through seignorage taxes (printing money) or fiscal taxes, and then transferring it to the aggrieved citizens. As all these forms of bailout amount to a redistribution of wealth from non-participants to participants, which is the critical assumption of this paper, the results of this paper are robust to how we model this redistribution.

[17] We make precise the phrase "too fast" in Appendix A.

1.2. The promoter of the Ponzi scheme

A risk-neutral promoter, who is outside the system, devises a classical Ponzi scheme. He has political connections, and this means that he is endowed with monopoly rights over the Ponzi scheme as well as the guarantee that the probability of intervention by the regulator, θ, is not unity. We do not model the rents he pays to the ruling political class to obtain these privileges.

In a round 0, the promoter sells certificates to a mass n_0 of citizens, promising them a return, r. So, if he prices these certificates at a price P per unit mass of citizens, he promises to redeem them for $P(1+r)$ in a round 1. In a round 1, he redeems these original certificates as promised by selling a fresh batch of certificates, again priced at P per unit mass, and a promise of a return r, to a mass equal to $n_1 = n_0(1+g)$ of citizens, where $g > r$.[18] And so on. Round L is the last round planned. The size of the population taking part in this last round is equal to $n_0(1+g)^L$. The five variables, r, g, P, n_0, and L, completely characterize the design of the Ponzi scheme, and are *endogenous decision variables* of the promoter.

We now look at the revenues of the promoter. From our above discussion, it is apparent that the mass of recruits is increasing at a constant growth rate every round, which implies $n_1 = n_0(1+g)$, $n_2 = n_1(1+g)$, and so on. In other words, the mass of citizens participating in a round i, $i = 0, 1, 2, \ldots, L$, is $n_0(1+g)^i$. The revenue of the promoter consists of the full amount of the sales achieved in the initial round, $n_0 P$, plus a fraction, t, of the revenue he raises every successive round till the very end. As the revenues raised in the later rounds are used to pay off the obligations of the previous round, it must mean that the revenue the promoter disburses in a round i, $(1-t)Pn_i = (1-t)Pn_{i-1}(1+g)$, must equal the obligation of the previous round, $P(1+r)n_{i-1}$. This means that t is not independent, but is given by the following accounting identity: $(1-t)(1+g) = (1+r)$, i.e., $t = (g-r)/(1+g)$. So the promoter's total expected revenue from the Ponzi scheme is

$$n_0 P + \big[(g-r)/(1+g)\big]\big[\{1-\theta\}^1 n_1 P + \{1-\theta\}^2 n_2 P + \cdots + \{1-\theta\}^{L-1} n_{L-1} P \\ + \{1-\theta\}^L n_L P\big],$$

where $\{1-\theta\}^i$ is the probability of survival of the Ponzi scheme till round L. The promoter's total expected revenue could be rewritten as

$$n_0 P + n_0 P\big[(g-r)/(1+g)\big]\Big[\big\{(1-\theta)(1+g)\big\}^1 + \big\{(1-\theta)(1+g)\big\}^2 + \cdots \\ + \big\{(1-\theta)(1+g)\big\}^{L-1} + \big\{(1-\theta)(1+g)\big\}^L\Big].$$

Note that the term in the second square bracket is the sum of a finite geometric series, a fact we will use later.

[18] Figure 1 shows a photocopy of a certificate issued by MMM in the Russian Ponzi scheme of 1994. The price of this certificate is 1000 roubles (shown on the front). Though it is true that this price, P, remained constant for every round, the promised r varied a little (the back shows the blank column on which the promised dividends were scribbled every round).

The spreading of news in this economy is modeled as follows. If n_i is the mass of citizens aware of the scheme in a round i, then $n_{i+1} = n_i D$ is the mass of citizens aware of the scheme in a round $i + 1$. The parameter D is greater than unity, and it is a measure of how "connected" the citizens of this economy are. The higher the D, the faster is the spread of news in the economy. This implies that more wired economies—wired in terms of communication linkages like the telephone or the Internet—have a higher D. As the pyramid cannot grow at a rate faster than the rate of spread of news, it immediately follows that

$$1 + g \leqslant D. \tag{1}$$

The only cost to the promoter is his direct marketing cost and the penalty he faces when the Ponzi scheme collapses. A distinguishing feature of Ponzi schemes is that the major marketing cost is the initial cost incurred to contact and convince citizens to buy into the scheme. Once a record of successful payment develops, and information about the fantastic scheme spreads, marketing costs to reach later recruits are negligible.[19] As a matter of fact, if marketing costs did not decrease as the number of rounds progress, there would be no reason to design these schemes as pyramids. For simplicity, we will assume that later marketing costs are zero. We adopt the following simple parameterization of the marketing cost function: the cost is $c + f(n_0)[n_0 P]$. The first term c is the fixed cost. The higher the c, the lower is the effectiveness of the mass media channel that is being used to reach the citizens. The second term is the variable cost, where $f(n_0)$ is the fraction of the initial-round revenue being paid out as a marketing commission: so $f(n_0) < 1$. Assume that $f(n_0)$ is increasing and convex in the initial mass of citizens contacted, n_0. The marketing literature on the effectiveness of advertising—see, for example, Rao and Miller (1975)—provides strong evidence in favor of this assumption. Finally, let d be the penalty the promoter faces when the Ponzi scheme collapses.

Table 1
Time line of the Ponzi scheme

0	1	2	3	…	$L-1$	L
n_0 contacted, by spending $c + f(n_0)n_0P$. Revenue $= Pn_0$	n_0D aware. Sell to $n_1 = n_0(1+g)$. Revenue $= Pn_1$. Use $(1-t)$ of revenue to pay off obligations of n_0	n_0D^2 aware. Sell to $n_2 = n_0(1+g)^2$. Revenue $= Pn_2$. Use $(1-t)$ of revenue to pay off obligations of n_1	n_0D^3 aware. Sell to $n_3 = n_0(1+g)^3$. Revenue $= Pn_3$. Use $(1-t)$ of revenue to pay off obligations of n_2	…	n_0D^{L-1} aware. Sell to $n_{L-1} = n_0(1+g)^{L-1}$. Revenue $= Pn_{L-1}$. Use $(1-t)$ of revenue to pay off obligations of n_{L-2}	n_0D^L aware. Sell to $n_L = n_0(1+g)^L$. Revenue $= Pn_L$. Use $(1-t)$ of revenue to pay off obligations of n_{L-1}

n_i is the mass of citizens that is to be contacted for the ith round, c is the fixed marketing commission, $f(n_0)$ is the fraction of the initial-round revenue being paid out as a variable marketing commission, P is the price of the certificate sold to each unit mass of citizens, D is the gross rate of the spread of news in the economy, g is the growth rate of the pyramid, and L is the total number of rounds to be played.

[19] According to Mackay (1841), thousands of working-class people crowded the streets of Paris in 1719 to buy shares as word about its success spread. To avoid the crowds, the bourgeoisie rented apartments near the temple of wealth. In 1994, this author saw huge lines in front of MMM's office in Moscow.

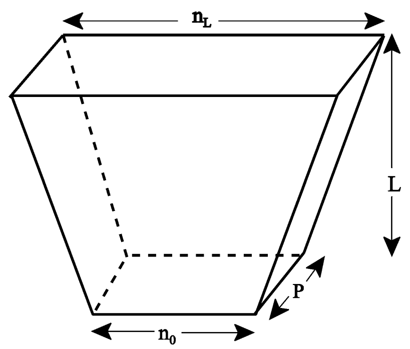

Fig. 3. The classic Ponzi scheme. n_i is the mass of citizens that is to be contacted for the ith round, P is the price of the certificate sold to each unit mass of citizens, and L is the total number of rounds to be played.

The regulator may intervene at any time with a probability θ. Table 1 gives the time line.

1.3. *The constrained optimization problem of the promoter*

The risk-neutral promoter's objective is to maximize expected profits in a round 0 by choosing five control variables: the promised return for each round, r, the growth rate of the pyramid, g, the price of the certificate sold to each unit mass of citizens, P, the mass of citizens that is to be contacted for the initial round, n_0, and the total number of rounds to be played, L. Note that L is the total number of rounds that the promoter is planning. The actual number of rounds played may be lower than L if the regulator intervenes before the planned termination.

Formally, the problem is defined as

$$\underset{\{r,g,P,n_0,L\}}{\text{Max}} \quad n_0 P\left[1+(g-r)(1-\theta)\frac{((1-\theta)(1+g))^L-1}{(1-\theta)(1+g)-1}\right] - \left[f(n_0)n_0P+c+d\right], \tag{2}$$

where the first square bracket multiplied by n_0P is the expected revenue, and the second square bracket is the marketing cost plus the penalty. While computing the expected revenue, we used the fact that the expected revenue of the promoter was the sum of a finite geometric series, $a + a^2 + \cdots + a^L$, whose sum is $a(a^L - 1)/(a - 1)$.

Except for r, the other control variables define the pyramidal shape of a classic Ponzi scheme. See Fig. 3 for a geometric representation.

The area of the base of the pyramid, n_0P, represents the revenue that the promoter collects in a round 0, and the area of the top of the pyramid, n_LP, represents the gross revenue that the promoter collects if round L is reached. Note also from the geometry of the pyramid that

$$n_L = n_0(1+g)^L. \tag{3}$$

1.4. The participation constraint of the promoter

Note that the promoter of a Ponzi scheme always faces a temptation of running away with the money before the planned termination in round L. If the promoter terminates in a round i, he gets a sure revenue of tPn_i, and if the promoter terminates at $i+1$, he gets an expected revenue of $(1-\theta)tPn_{i+1} = (1-\theta)t(1+g)Pn_i$. To insure that it is optimal for the promoter to wait till round L, i.e., that the Ponzi scheme is subgame-perfect, the latter expected revenue should be greater than or equal to the former sure revenue. The participation constraint of the promoter, therefore, gives us the following lower bound on g:

$$g \geqslant \frac{\theta}{(1-\theta)}. \tag{4}$$

1.5. The participation constraint of the citizens

The participation constraint of the citizens depends on our assumption of the information distribution in the economy. We will examine a symmetric information environment in this paper. We will assume that as soon as a citizen becomes aware of a Ponzi scheme, he knows when the scheme began, which round is being played, and when it is planned to end.[20]

Two more simplifying assumptions are needed to make the problem tractable. First, we will assume that aware citizens can participate once every round, for as many rounds as they like. So a unit mass citizen is restricted to just one certificate per round. This means that the aware unit mass citizen can only invest P in the Ponzi scheme every period. This assumption is needed because if we do not restrict the demand, risk-neutral agents will either not participate or will want to invest an infinite amount of their money.[21] So, with some loss of generality, we assume that unit mass citizens are restricted to investing a maximum of, say P, every period.[22] The second assumption we need to make is that of a

[20] This seems like a severe assumption. It is. As discussed in the introduction, previous literature on rational bubbles in a finite economy has always assumed asymmetric information. By making the assumption of symmetric information, we are stacking the odds against finding a rational Ponzi scheme in a finite economy. If the promoter can devise rational Ponzi schemes in a finite economy under scenarios where the investors know exactly which round they are playing, it is easier for the promoter to devise rational Ponzi schemes in a finite economy where investors do not know which round they are playing. In a previous version of this paper, we had relaxed the assumption that the citizen knew which round he was playing. This made the model more realistic. However, this benefit of realism came at the cost of model complications, which distracted from the simple insights of this paper.

[21] A similar argument is made in market microstructure models to restrict agents to buy or sell just one share (see Glosten and Milgrom (1985)).

[22] Figure 2 shows a photocopy of a certificate issued by the Gjallica foundation in Albania. It promises a monthly interest rate of 10% (214% annualized) for a holding period of 6 months (October 2, 1996 to April 2, 1997). A clause in the contract stipulates that at the end of the contracted time, the lender can withdraw the principal and interest, or *roll it over*. Our assumption of restricting investment per round per person to a maximum of P seems to imply that we are precluding rollovers. That is not true. Our model can easily accommodate rollovers as long as we assume that the total revenue (price per certificate multiplied by the mass of citizens participating) grows at a constant rate g. This means that if we allow rollovers, price will grow at a constant rate r

tie-breaking rule. We will assume that citizens participate in the Ponzi scheme if they are indifferent between participating and not participating.

2. Designing the optimal Ponzi scheme

Citizens know which round they are playing. The citizens also realize that though L is the number of rounds planned, the Ponzi scheme may be terminated before L by the regulator. The promoter, on the other hand, given inequality (4), would not like to terminate early. Under these assumptions, let us now design the optimal Ponzi scheme for the promoter.

The objective of the promoter is to maximize his expected profits given in (2), subject to the constraint that the citizens take part in each round i, where $i = 0, 1, 2, \ldots, L$. Notice that once a citizen becomes aware of the Ponzi scheme, he does not forget it. This means that if the promoter has set up his scheme such that it provides incentives for a citizen to participate in a round i, the citizens who become aware of the scheme in a round i will participate, as well as the citizens who became aware of the scheme before a round i. In other words, once a citizen becomes aware and chooses to participate, he will choose to participate in successive rounds as well.

2.1. The participation constraint of the citizens

We solve backwards. Assume we have reached round L. The promoter has planned to terminate the Ponzi scheme in this round. A mass of n_L citizens have been contacted. Some of them have just become aware of the Ponzi scheme, and some have been aware from previous rounds. Will they participate, knowing that they will lose their investment P for sure? They may or may not be bailed out, the probability of the bailout being $p(n_a)$, where $n_a = n_L$ is the mass of citizens affected. We get our first lemma.

Lemma 1. *The promoter of a Ponzi scheme plans to involve at least a mass n^* of citizens in the last round, i.e.,*

$$n_a = n_L \geqslant n^*. \tag{5}$$

The proof is as follows. Given the political regimes under consideration, there is no bailout if the mass of citizens involved is less than the critical mass n^*. If there is no bailout, participation guarantees a sure loss of P for the unit mass citizen, whereas non-participation ensures no loss/no gain. So no one will participate in the last round. If no one participates in the last round, by backward induction, no one will participate in any round. This leads to an obvious corollary.

Corollary 1. *Ponzi schemes do not exist if the probability of a bailout is zero.*

(instead of remaining fixed) and the number of participants will be restricted to grow at $(g - r)/(1 + r)$ (instead of growing at g).

This does not mean that Ponzi schemes necessarily exist when the probability of a bailout is positive; we need more conditions for that.

Given that the mass of citizens in the last round is at least n^*, if a unit mass citizen participates in the last round, his expected payoff is

$$-P + \alpha(1/n_a - 1)\beta. \tag{6}$$

It is his sure loss of P ameliorated somewhat by the expected gain from the redistribution inherent in the bailout. The expected payoff of a unit mass citizen if he does not participate in the last round is

$$-\alpha\beta. \tag{7}$$

This is his expected loss from the redistribution inherent in the bailout. So the citizen will participate in the last round if (6) $\geqslant$ (7), which gives us the following condition on the price of the certificate:

$$P \leqslant \frac{\alpha}{n_a}\beta. \tag{8}$$

An important point needs to be made here. The citizen participating in the last round has a difficult choice: if he participates, he loses, but he loses less than what he would if he did not participate. This dramatizes the main insight of this paper: it is *rational* for citizens to take part in spectacular Ponzi schemes that are certain to explode because they know that, since so many people are taking part, bailouts are probable, and bailouts are inherently a redistribution of wealth from non-participants to participants. It is now also clear why it is in the public interest to stop this scheme, but because citizens are atomistic and there exists no representative government, there is a coordination failure.

What are the participation constraints of the citizens in the second-last round? A citizen who becomes aware of the scheme in the second-last round will continue to be aware of the scheme when the last round comes. Condition (8) had ensured that he will participate in this last round. So the only question to answer is whether he will participate in the second-last round. His expected profit from participating in the second-last round is:

$$\big[(1-\theta)Pr + \theta(-P)\big], \tag{9}$$

where the first expression in (9) is his net profit from investment in this round if the regulator does not terminate the scheme (which occurs with a probability $1-\theta$) and the second expression in (9) is his loss if the regulator does terminate the scheme (which occurs with a probability θ). He will participate if (9) $\geqslant 0$. This gives us the following lower bound on r:

$$r \geqslant \frac{\theta}{(1-\theta)}. \tag{10}$$

Using the same logic, it is easy to demonstrate that there will be participation by the citizen in all the earlier rounds as well.

2.2. The participation constraint of the promoter

From (2), it is apparent that the expected profit of the promoter monotonically increases with P. Hence, for maximum expected profit, the constraint on P given by (8) becomes binding. This gives us:

$$P = \frac{\alpha\beta}{n_a}. \tag{11}$$

In the previous section we had showed that it is not optimal for the promoter to terminate the scheme before round L. We now address why the promoter intends to terminate at round L, and not terminate later. The reason is the following. Let n_a be the mass of citizens that are aggrieved when the promoter terminates the Ponzi scheme. If the promoter terminates at round L, he gets a sure revenue of tPn_a, and if the promoter terminates at round $L+i$, where $i = 1, 2, \ldots$, he gets an expected revenue of $t(1-\theta)^i Pn_a$. Substituting from (11), it is easy to check that if the promoter terminates at L, his sure revenue is equal to $t\alpha\beta$, and if the promoter terminates at $L+i$, where $i = l, 2, \ldots$, his expected revenue is equal to $(1-\theta)^i t\alpha\beta$. The former sure revenue is greater than the latter expected revenue. So it is optimal to terminate at L.

2.3. The optimal Ponzi scheme

We are now in a position to solve the optimal control problem of the promoter. Substituting for P from (11), for n_a from (5), and for L from (3) in (2), the objective of the promoter reduces to

$$\max_{\{r,g,n_0,n_L\}} \quad n_0 \frac{\alpha\beta}{n_L}\left[1 + (g-r)(1-\theta)\frac{((1-\theta)(1+g))^{(\ln n_L - \ln n_0)/\ln(1+g)} - 1}{(1-\theta)(1+g) - 1} - f(n_0)\right] - [c+d]. \tag{12}$$

Note that we have replaced L by n_L as a control variable above. This is possible given equality (3). The objective function (12) reveals three interesting observations. First, holding the other control variables constant, expected profit of the promoter is decreasing in r (this is obvious from (12)), increasing in g (this is not obvious from (12) and, therefore, formally proved in Appendix A), and is decreasing in n_L (this is not obvious from (12) and, therefore, formally proved in Appendix A).

The intuition is as follows. As the promoter wants to retain as much revenue as possible each round, he wants to increase t, which means that he wants to increase the wedge between g and r. So he wants to increase g and decrease r. He sets g at its upper bound and r at its lower bound. So, from (1),

$$g = D - 1, \tag{13}$$

and, from (10),

$$r = \theta/(1-\theta). \tag{14}$$

The upper bound of g is a function of D. This is because the growth of the pyramid, g, is constrained by how fast the news moves in the economy. The lower bound of r is a function of θ. This is because the interest rate r should provide just enough incentives for the citizens to take part in each round. To be precise, the citizen will take part if his net profit from participating in all rounds except the last round, $(1-\theta)Pr+\theta(-P)$, is non-negative. This gives us the lower bound on r: $(1-\theta)(1+r)\geqslant 1$, i.e., $r\geqslant\theta/(1-\theta)$. Equation (14) tells us that the higher the probability of termination of the Ponzi scheme by a regulator, the higher is the return offered in a Ponzi scheme. This is a testable implication.[23]

The reason expected profits are decreasing in n_L is because expected profits decrease as the price per certificate decreases, and the price per certificate decreases as n_L increases. The reason the price per certificate decreases as n_L increases is because rational agents will only take part if their loss—the price per certificate—is compensated fully by the expected gross bailout per person, and the expected gross bailout per person decreases as the mass of persons receiving the bailout, n_L, increases. So the promoter sets n_L at its lower bound. So, from (5),

$$n_L = n^*. \tag{15}$$

The optimal P, from (5), (11), and (15) is:

$$P = \frac{\alpha\beta}{n^*}. \tag{16}$$

Substituting for n_L from (15), for r from (14), for g from (13) in (12), the promoter's objective reduces to

$$\begin{aligned}&\operatorname*{Max}_{\{n_0\}}\ \left\{\frac{\alpha\beta}{n^*}k(n_0)\right\}-\{c+d\},\\ &\quad\text{where}\quad k(n_0)\equiv n_0\left[-f(n_0)+\big((1-\theta)D\big)^{(\ln n^*-\ln n_0)/\ln D}\right].\end{aligned} \tag{17}$$

The optimal value of n_0 is solved in Appendix A. We state it below for completion.

$$\begin{aligned}n_0\text{ solves}\quad k'(n_0)&\equiv -n_0 f'(n_0)-f(n_0)+\big((1-\theta)D\big)^{\frac{\ln n^*-\ln n_0}{\ln D}}\left(-\frac{\ln(1-\theta)}{\ln D}\right)\\ &=0.\end{aligned} \tag{18}$$

Proposition 1. *A Ponzi scheme may exist if an economy has a large public sector* (α *is bounded below*)*, and the assets of the state could be used for a bailout* (β *is bounded below*)*, and the probability of early termination of the Ponzi scheme by a regulator is low* (θ *is bounded above*)*, and there is inexpensive access to citizens through the mass media* (c *is bounded above*)*, and there are no severe penalties on promoters of Ponzi schemes* (d *is bounded above*).

[23] We offer this corroborating anecdotal evidence: the promoters in Albania in 1997 were more politically connected than the ones in Russia in 1994; the Albanian Ponzi schemes offered much lower returns than the Russian Ponzi schemes.

We prove Proposition 1 intuitively. Note from (18) that n_0 is a function only of the deep parameters n^*, D, and θ; note from (15) that n_L is a function only of the deep parameter n^*; note from (13) that g is a function only of the deep parameter D; note from (14) that r is a function only of the deep parameter θ. Therefore L, from the geometry of the pyramid scheme (3), is also a function of only the deep parameters n^*, D, and θ. In particular, only P is affected as α or β changes. As α decreases or β decreases, from (16), P decreases. This means, from (2), that the maximum expected profits of the promoter decrease. The maximum expected profit also decreases as c increases or d increases. This implies that if α or β is below a lower bound, or c or d is above an upper bound, the maximum expected profits of the promoter are negative, and so he would not initiate a Ponzi scheme. The upper bound for θ comes because of the following reason. From (13) and (14), we know that $\theta/(1-\theta) = r < g = D - 1$, which implies that $\theta \leqslant (D-1)/D$.

Note that if the condition in Proposition 1 is satisfied, the optimal Ponzi scheme will have the following design features: the promised interest rate, r, will be $\theta/(1-\theta)$; the growth rate of participants, g, will be $D - 1$; the price per certificate, P, will be $(\alpha\beta)/n^*$, the initial mass of participants, n_0, will be as given in (18); and the number of rounds planned, L, will be $(\ln n^* - \ln n_0)/\ln D$.

Proposition 1 leads to two interesting points. First, it seems that the political economies where Ponzi schemes can exist are likely to be transition economies. As discussed in the introduction of this paper, such economies may meet the five requirements mentioned in Proposition 1. Second, as $\theta/(1-\theta) = r < g = D - 1$, the upper bound of the parameter θ is $(D-1)/D$. This upper bound increases as D increases. This means that in an economy that becomes more "wired," i.e., when D increases, the probability of intervention by the regulator has to increase, i.e., θ has to increase.

The second point has profound implications for the regulation of investment proposals over the Internet. Though it is unambiguous about its policy prescription—regulators need to become more vigilant in their examination of investment schemes as society gets more wired—it is very important to point out the reason which drives this policy prescription. The reason is that, as a society gets more wired and it is easier to contact more people, the "too big to fail doctrine" becomes easier to exploit. Hence, we either need more regulatory vigilance or we need more statements of caveat emptor.

No government has ever compensated its citizens fully for what they have lost in Ponzi schemes. A belief that such an occurrence is likely is unreasonable. This leads us to the last result of this paper.

The money the unit mass citizen loses when the Ponzi scheme collapses is P. This equals, from (16), $(\alpha\beta)/n^*$. The net bailout for a unit mass citizen who has participated in a Ponzi scheme that has exploded is $\alpha(1/n^* - 1)$. Therefore, the fraction of the money lost that is recovered by a bailout, defined as δ, is

$$\delta = \frac{\alpha(1/n^* - 1)}{\alpha\beta/n^*} = \frac{(1-n^*)}{\beta}. \tag{19}$$

A partial bailout implies that $\delta < 1$. From (19), if $(1-n^*) < \beta$, then $\delta < 1$. This leads us to Proposition 2.

Proposition 2. *A Ponzi scheme will exist even under partial bailout, if the condition in Proposition* 1 *holds and the probability of bailout,* β*, is higher than* $(1 - n^*)$*, where* n^* *is the critical fraction of citizens that are required to be involved for there to exist the possibility of a bailout.*

3. Conclusion

This paper is motivated by the plethora of Ponzi schemes that erupted in transition economies in the 1990s. The commentary at that time suggested that these Ponzi schemes were happening because citizens in these economies, not being used to the subtleties of capitalism, believed that they could achieve high expected returns without taking the commensurate risk.[24]

We propose an alternative hypothesis in this paper. Our hypothesis is based on the assumption that in a transition economy where there is no representative government, where property rights may be transferred from the state to its citizens in an unclear manner, and where interest groups are beginning to have a political voice, it is not foolish for the citizen to believe that the state assets *may* be used *to partially* bailout a large aggrieved group of citizens. This has *happened before* in history.

Given this belief of a bailout, which is tantamount to a redistribution from non-participants to the aggrieved participants of a failed Ponzi scheme, this paper provides an example to show how an unscrupulous profit-maximizing promoter can exploit this belief to convince citizens to participate in Ponzi schemes that are certain to explode.

The paper formalizes the particular political economy where this might happen. There should be a large public sector (the proportion of national wealth held by the state is above a lower bound), ambiguous laws governing the transfer of property rights from the state to the citizen (victims of a failed Ponzi scheme may organize to use the state's assets for a bailout, the probability of which to occur is above a lower bound), weak law enforcement (the probability of early termination of the Ponzi scheme by the state is below an upper bound), an inexpensive access to citizens through mass media (media cost is below an upper bound), and little punishment for the promoter of the Ponzi scheme (penalty is below an upper bound).

If such a political economy exists, we go on to completely characterize the Ponzi schemes that will occur. We develop closed-form expressions for the five defining variables of a classic Ponzi scheme: the promised return for each round, the growth rate of the pyramid, the price of the certificate sold to each unit mass of citizens per round, the mass of citizens that is to be contacted for the initial round, and the total number of rounds to be played.

The limitation of this paper is that we restrict our attention to simple Ponzi schemes under simple information scenarios. By restricting ourselves to time-invariant parameters, we preclude all interesting dynamics. In actual Ponzi schemes, citizens learn as the rounds

[24] See Bezemer (1998) for this point of view.

progress, and the promoter alters his control parameters accordingly. These limitations we leave for future research to rectify.

We conclude with a discussion on the welfare aspects of Ponzi schemes and state bailouts. It is a fact that all governments are always engaged in some form of Ponzi scheme. Not only are governments conducting these schemes, but they are also insuring their monopoly by banning other entrants. Fiat money and social security are relevant examples. Is this good or bad? A major contribution of the past literature, where many different types of overlapping generation models have been analyzed, has been to show that some monopolistic Ponzi schemes run by the state (for example, fiat money) could be welfare-improving. As for state bailouts, these are an omnipresent feature of all economies. Bank bailouts are relevant examples. Are these good or bad? Again, the banking literature has shown that these may be needed to prevent a systemic collapse of the banking sector.[25] What about the Ponzi schemes and bailouts analyzed in this paper? The answer is unequivocal: bailouts are a bad idea. Unlike other types of economic crises for which a case for a bailout can be made—for example, a banking crisis—bailouts of citizens who have lost their money in Ponzi schemes amount to compensating people for their foolish decisions. It is "pure" moral hazard, and it will lead to more foolish decisions. Besides, since the scheme is really a cynical exploitation of the "too big to fail doctrine" by a citizen usurping the powers of the state, bailouts would encourage such people.

This paper then suggests a simple solution to prevent Ponzi schemes: the citizen should not be led to believe that there ever will be a bailout. However, considering that ex-ante promises of no bailout are not credible ex-post even for well-developed capitalist economies, it is even less credible in transition economies where, ironically, three progressive forces—the ability to advertise, the growth of telecommunications and burgeoning "people power"—exacerbate the problem. We conclude with this sobering thought.

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[25] The case for bailouts was formalized in the classic Diamond and Dybvig (1983) paper. Demirguc-Asli and Kane (2002) provide a comprehensive survey of deposit insurance around the world.

Economies, provided many thoughtful comments. I am particularly grateful to Mukarram Attari, Gary Gorton, Bryan Routledge, Javier Suarez, and an anonymous referee for their insights. All errors are my own.

Appendix A

Note from (12) that it is apparent that expected profit is decreasing with r. So, if we substitute the minimum value of r, which is given in (14), in (12), we obtain, after some simplification:

$$\frac{n_0\alpha\beta}{n_L}\Big[\big((1-\theta)(1+g)\big)^{(\ln n_L - \ln n_0)/\ln(1+g)} - f(n_0)\Big] - [c+d]. \tag{A.1}$$

Proof that expected profit increases with g

The first derivative of (A.1) with respect to g, after simplification, is:

$$\begin{aligned}&\frac{n_0\alpha\beta}{n_L(1+g)}\Big(\big((1-\theta)(1+g)\big)^{(\ln n_L - \ln n_0)/\ln(1+g)}\Big)\left(\frac{\ln n_L - \ln n_0}{\ln(1+g)}\right)\\ &\quad\times\left(\frac{-\ln(1-\theta)}{\ln(1+g)}\right).\end{aligned} \tag{A.2}$$

As $\theta < 1$, (A.2) > 0. So expected profit increases with g. Q.E.D.

The derivation of the optimal n_0 and the proof that expected profit decreases with n_L

We have already shown that the expected profit increases with g. If we substitute the maximum value of g, which is given in (13), in (A.1), the objective function of the promoter simplifies to:

$$\begin{aligned}&\operatorname*{Max}_{\{n_0\}}\ \left\{\frac{\alpha\beta}{n_L}k(n_0)\right\} - \{c+d\},\\ &\quad\text{where}\quad k(n_0) \equiv n_0\Big[-f(n_0) + \big((1-\theta)D\big)^{(\ln n_L - \ln n_0)/\ln D}\Big].\end{aligned} \tag{A.3}$$

(A.3) is the same as (17) in the text, with n_L instead of n^*. The first derivative of (A.3) with respect to n_0 is:

$$\begin{aligned}&\frac{\alpha\beta}{n_L}\left[-n_0 f'(n_0) - f(n_0) + \big((1-\theta)D\big)^L + n_0\big((1-\theta)D\big)^L \ln\big((1-\theta)D\big)\frac{\mathrm{d}L}{\mathrm{d}n_0}\right]\\ &\quad\equiv \frac{\alpha\beta}{n_L}k'(n_0).\end{aligned} \tag{A.4}$$

From (3) and (13),

$$\frac{\mathrm{d}L}{\mathrm{d}n_0} = -\frac{1}{n_0 \ln D}. \tag{A.5}$$

Substituting (A.5) in (A.4), the first derivative simplifies to

$$\frac{\alpha\beta}{n_L}k'(n_0) \equiv \frac{\alpha\beta}{n_L}\left[-n_0 f'(n_0) - f(n_0) + \left\{\left(\left((1-\theta)D\right)^{(\ln n_L - \ln n_0)/\ln D}\right)\left(-\frac{\ln(1-\theta)}{\ln D}\right)\right\}\right]. \tag{A.6}$$

The f.o.c. is the value of n_0 that makes (A.6) equal zero. This gives us (18) in the text, with n_L instead of n^*. The second derivative of (A.3) with respect to n_0, i.e., the first derivative of (A.6) with respect to n_0 is

$$\frac{\alpha\beta}{n_L}k''(n_0) \equiv \frac{\alpha\beta}{n_L}\left[-n_0 f''(n_0) - 2f'(n_0) - \left(\left((1-\theta)D\right)^{(\ln n_L - \ln n_0)/\ln D}\right) \times \left(\frac{\ln(1-\theta)D)}{n_0 \ln D}\right)\left(-\frac{\ln(1-\theta)}{\ln D}\right)\right]. \tag{A.7}$$

As $f'(\cdot)$ and $f''(\cdot)$ are positive, $1+g = D > (D(1-\theta)) > 1$ and $\ln(1-\theta) < 0$, the expression (A.7) is negative. So this is a well-defined maximization problem. Q.E.D.

The first derivative of (A.3) with respect to n_L, after simplification, is:

$$-\frac{n_0\alpha\beta}{n_L^2}\left[-f(n_0) + \left\{\left(\left((1-\theta)D\right)^{(\ln n_L - \ln n_0)/\ln D}\right)\left(\frac{-\ln(1-\theta)}{\ln D}\right)\right\}\right]. \tag{A.8}$$

The optimal value of n_0 is that n_0 which makes (A.6) equal zero. If (A.6) equals zero, then $n_0 f'(n_0) + f(n_0)$ equals the expression in curly brackets in (A.6). But the expression in curly brackets in (A.6) is the same as the expression in curly brackets in (A.8). Also, $n_0 f'(n_0) + f(n_0) > f(n_0)$. So (A.8) is negative. This implies that the expected profit of the promoter is decreasing in n_L. So the optimal n_L is its minimum value which from (5) is n^*. This proves (15). Further, n_0 has an interior solution. The optimal value of n_0 is that n_0 which makes (A.6) equal zero with n_L being replaced by n^*. That proves (18). Q.E.D.

Generalizing α to $\alpha(n_L)$

Suppose we make the more realistic assumption that the bailout size will depend on the numerical strength of the aggrieved citizens, i.e., α is an increasing function of n_L. Will our results go through? The derivative of (A.3) with respect to n_L with this more general formulation is:

$$-\frac{n_0\alpha(n_L)\beta}{n_L^2}\left[-f(n_0) + \left\{\left(\left((1-\theta)D\right)^{(\ln n_L - \ln n_0)/\ln D}\right)\left(\frac{-\ln(1-\theta)}{\ln D}\right)\right\}\right] + \frac{n_0\alpha'(n_L)\beta}{n_L}\left[-f(n_0) + \left\{\left(\left((1-\theta)D\right)^{(\ln n_L - \ln n_0)/\ln D}\right)\right\}\right]. \tag{A.9}$$

Notice that (A.9) can still be negative, and the optimal n_L will continue to be its minimum of n^*, if $\alpha'(n_L)$ is bounded above. The upper bound can be formally computed by solving for the $\alpha'(n_L)$ that makes (A.9) equal to zero. Q.E.D.

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